

BIM / First Semester / MTH 204: Basic Mathematics

Candidates are required to answer the questions in their own words as far as practicable.

Group "A"

[10 × 2 = 20]

Brief Answer Questions:

1. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$, find $(A \cap B)$.
2. Express the complex number $\frac{2-\sqrt{-25}}{1-\sqrt{-16}}$ into the form $a + ib$.
3. Solve the inequality $|3x - 1| \leq 7$.
4. Evaluate: $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$.
5. Find a vector of length 3 having direction same as the direction of vector $\vec{a} = 5\vec{i} - 4\vec{j}$.
6. Find $\frac{dy}{dx}$ of $y = \frac{1}{\sqrt{3x+1}}$.
7. If $P = \begin{bmatrix} -1 & 0 & 2 \\ 4 & 0 & -5 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 1 \\ -2 & 2 \\ 0 & 3 \end{bmatrix}$, find $5PQ$.
8. Find the area bounded by the curve $y = 3x^2 - 2$, x-axis and the ordinates at $x = 1$ and $x = 2$.
9. Solve the differential equation: $\frac{dy}{dx} = 5y$.
10. Evaluate: $\begin{vmatrix} 2 & -3 & 4 \\ -1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix}$.

Group "B"

Short Answer Questions: (Attempt any SIX Questions)

[6 × 5 = 30]

11. (a) Find the square roots of $4 + 3i$.
(b) If ω is an imaginary cube roots of unity then prove or disprove that $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 = 0$.
12. A function $f(x)$ is defined by $f(x) = \begin{cases} \frac{2x^2 - 18}{x - 3} & \text{for } x \neq 3 \\ k & \text{for } x = 3 \end{cases}$
Find the value of k so that $f(x)$ is continuous at $x = 3$.
13. Prove that: $\lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{x+a} - \sqrt{x}) = \frac{a}{2}$.
14. Find the derivative of:
(a) $y = \frac{1}{\sqrt[3]{6x^5 - 4x^3 + 7}}$ (b) $y = \frac{e^{2x}}{\log x}$

15. Evaluate the integrals:

(a) $\int \frac{x^2}{x+1} dx$

(b) $\int \frac{(1+\log x)^3}{x} dx$

16. If $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$ are pair of vectors then

(a) Find $\vec{a} \times \vec{b}$

(b) Find a unit vector perpendicular to $\vec{a}$ and $\vec{b}$

(c) Show that $\vec{a} \times \vec{b}$ is perpendicular to the vectors $\vec{a}$ and $\vec{b}$

17. Solve the differential equation: $(x^2 - 1) \frac{dy}{dx} + 2xy = 1$

Group "C"

Long Answer Questions: (Attempt any THREE Questions)

[3 × 10 = 30]

18. In a group of students 18 read marketing, 22 read statistics and 16 read economics, 6 read marketing only, 9 read statistics only, 5 read marketing and statistics only and 5 read statistics and economics only.

a. How many read all the subjects?

b. How many read marketing and economics only

c. How many read economics only?

d. How many students are there altogether?

Represent all the sets in venn diagram.

19. The following table shows the yearly population of the city.

Year	2016	2017	2018	2019	2020	2021	2022
Population (in million)	23	24	28	31	33	35	38

Obtain the equation of the line by least square method. Also estimate the population of the city in 2023 and 2024.

20. A transport company has three types trucks A, B and C which are designed to carry three different sizes of boxes P, Q and R per load as shown.

Boxes	Type of trucks		
	A	B	C
P	2	5	2
Q	3	2	5
R	1	9	0

How many trucks of each type should be used to carry exactly 18 boxes of size P, 18 boxes of size Q and 21 boxes of size R? (use cramer's rule or inverse matrix method)

21. The demand and supply functions of a good under perfect competition are given by $P_d = 240 - 0.6 Q_d$ and $P_s = 80 + 0.2 Q_s$ respectively, where P and Q denote price and quantity. Find the consumer's surplus, producer's surplus and total surplus.

Group "D"

Comprehensive Answer / Case / Situation Analysis Questions:

[20]

22. If the demand function of monopolist is given $P = 100 - Q$ and its cost function is given by $C(Q) = 100 + 10Q + 4Q^2$.

- a. Find the value of Q at which the revenue is maximum. Also find the maximum revenue.
- b. Find the value of Q at which profit is maximum. Also find the maximum profit.
- c. Find the price (P) at which profit is maximum
- d. Also, determine the intervals at which profit function is increasing or decreasing.

[6+6+2+6]

